

A Pie That Can't Be Cut Fairly

Walter Stromquist
Swarthmore College
mail@walterstromquist.com

Abstract for Dagstuhl Conference on Fair Division
June 19, 2007

David Gale asked in 1993 whether, when a pie is to be divided among n claimants, it is always possible to find a division that is both envy free and undominated (Pareto optimal). The pie is cut along n radii, and the claimants' preferences are described by different measures on the cake.

We answer Gale's question in the negative for $n = 3$ by exhibiting three measures for which no division of the pie can be both envy free and undominated. The measures are absolutely continuous with respect to each other and with respect to area.

The measures can be described by partitioning the pie into 18 sectors and specifying the value assigned by each claimant to each sector:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
<i>A</i>	130	100	69	131	70	100	100	70	131	69	100	130	60	100	100	100	100	60
<i>B</i>	100	100	115	5	100	100	100	100	100	100	100	100	100	100	100	100	100	100
<i>C</i>	100	100	100	100	100	100	100	100	115	5	100	100	100	100	100	100	100	100

Each measure is uniform on each sector. The key to the proof is that envy-free allocations in which the total of the values to the players does not exceed 1800 are dominated, while allocations with totals above that value are not envy free.